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Notes on experimental velocity profiles in laminar flow around spheres at intermediate Reynolds numbers

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Seeley, Hummel & Smith (1975) reported the results of experiments to study the dynamics of flow around spheres at intermediate Reynolds numbers using a non-disturbing flow-visualization technique. The flow patterns were recorded on ciné photographs and the information stored was processed in order to obtain the velocity field. The position of fluid elements shown by the photochromic indicator traces were estimated by eye on a projection screen. In this paper, a new set of results based on the same films has been reduced and computed using the 'POLLY' film-reading system described by Esmail, Smith & Hummel (1976). Some numerical boundary-layer solutions are included to show the reliability of the data, and comparisons with the results previously reported by Seeley *et al.* (1975) are presented.‡

The experimental method was described in detail by Popovich & Hummel (1967*a, b*), Smith & Hummel (1973) and Seeley *et al.* (1975). The basic idea is to introduce a dark trace in a transparent liquid flow. This dark trace is formed as a result of the reaction of the photochromic indicator to an intense ultraviolet-light flash. The light sources used are generally lasers. The movement of the dark trace is recorded on ciné photographs. Each subsequent series of frames contains information describing the subsequent positions and shapes of the trace related to time. The reduction of the experimental information recorded on film to a velocity field goes through various stages of film reading and data processing.

In the work of Seeley *et al.*, the experimental film data were read by eye as follows. The film was projected frame by frame onto a screen. The x, y co-ordinate data were read by eye from a projection on a screen with 1×1 cm grid, and the readings transferred to computer cards. The precision of this vital procedure depended mainly on the visual ability of the operator. This reading error, i.e. the error arising when the centre-line of a finite trace is determined by eye, as discussed by Seeley *et al.* (1975), could have been reduced by repeated re-reading, but this would have been tedious in the extreme.

Recently, a film-reading computer-controlled system was employed to re-analyse the data of Seeley *et al.* Its main component is an electronic film-scanning measuring device, controlled by a computer. The link to the computer permits input of elaborate scanning, filtering and processing algorithms, which automatically control the reading process. The purpose is to eliminate the reading error mentioned above. Direct access to computer storage is an additional advantage of the system. The film-reading computer system and its application to the flow-visualization technique are described by

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Esmail *et al.* (1976). Undoubtedly, the results obtained using this electronic automatic scanning device are far more reliable than those obtained by eye; the computer control virtually eliminates the reading error and operator bias.

The reduction of the photochromic tracer films to numerical data is followed by data processing to obtain the velocity field around the spheres. This procedure is based on the conservation of mass, which in incompressible two-dimensional laminar flow leads to the conservation of projected areas. Seeley *et al.* (1975) used a complicated iterative procedure, covering the entire sphere region at once to obtain the tangential and radial velocities. The trace positions were fed to the computer as initial data, and the iterative procedure eventually led to the reported velocity field. Such a method is apt to introduce some systematic errors into the original experimental data. However, it was not followed by a comparison of the resultant velocity field with the positions of the initial trace, to make sure that such errors, if any, were minimal. In this work, doubts about the introduction of such modifications were eliminated by using a very simple straightforward data-processing method. The principle is that any amount of liquid bounded by a combination of streamlines and a solid surface moves to an area equal to the starting area. The radial and tangential velocities were determined at each point independently, without iterations involving the velocities at any other point. In order to avoid error accumulation, the procedure was designed to start at any point of the field. Therefore the truncation error in the values determined is independent of the entire field, and may be assumed to be uniform across the flow.

Results and discussion

Since the new set of results is based on the films containing the experimental data of Seeley *et al.* (1975), it is important to compare it with the results reported by these authors to point out the corrections introduced here. The overall impression is that, in spite of the relatively poor data-reduction technique of Seeley *et al.*, there is no general qualitative disagreement between their results and the results obtained by the more sophisticated technique of the present work. As might be expected, however, the reading error and the iterative method of Seeley *et al.* (1975) led to quantitative differences between the two sets of results.

A typical plot of the tangential velocities is given in figure 1 and shows that the velocities obtained by Seeley *et al.* and in the present study tend to differ systematically away from the front stagnation point. Although the POLLY film-reading system used in the present work is more accurate than the technique used by Seeley *et al.* (1975), the human eye is capable of some interpolation and extrapolation in cases when the photochromic trace is fading out. Because of this, the present results could not be extended to an angle of 2 rad as in the work of Seeley *et al.* Figure 2 displays a typical plot of the radial velocities in which the results of the present work are compared with those of Seeley *et al.* Since, in general, the magnitude of this component is small, it is expected to be more sensitive to error. The results obtained in this work (curve 1) show a definite kink in a typical radial velocity curve at about 60° from the front stagnation point. This kink is not apparent from the work of Seeley *et al.* (curve 2). No explanation is available to explain this effect, which may be been introduced by the sphere support. For the same reason mentioned before, the present results could not be extended to an angle of 2 rad.

As was shown by Seeley *et al.*, the measured tangential velocity develops a maximum

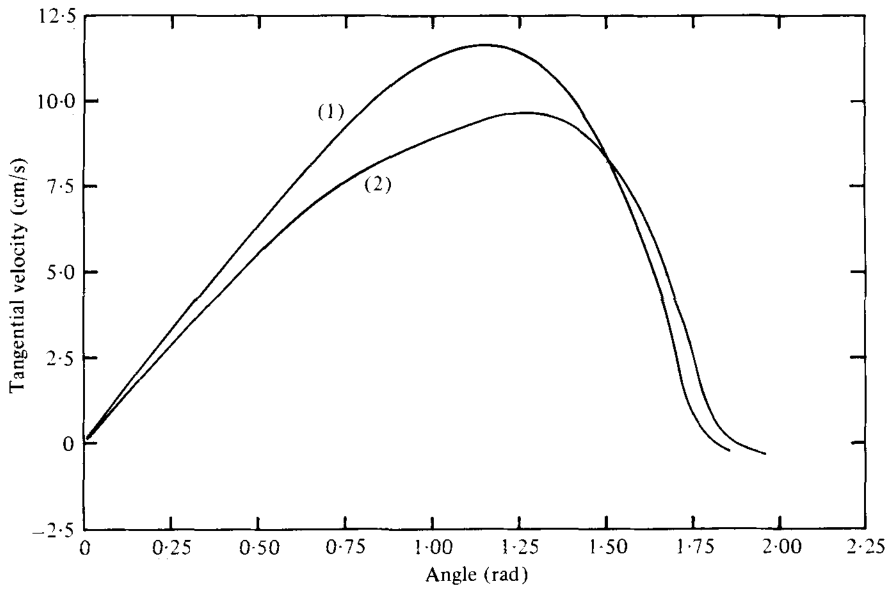

FIGURE 1. Tangential velocity at a radial distance $(Y-R)/R = 0{\cdot}063$. (1) This work. (2) Seeley *et al.*

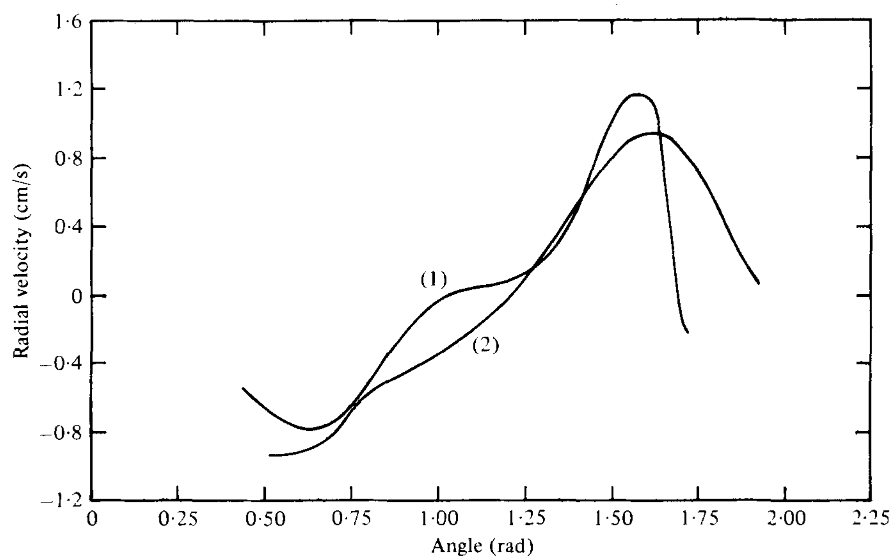

FIGURE 2. Radial velocity at radial distance $(Y-R)/R = 0{\cdot}063$. (1) This work. (2) Seeley *et al.*

value in the boundary layer greater than the free-stream component but less than that predicted by potential flow. Curve 3 in figure 3 shows the surface vorticity calculated numerically by the method of Frossling (1958), in which the maximum measured velocity U_M is used as the outer boundary condition. Curve 4 shows the result of the same calculation with U_M replaced by the commonly applied potential-flow maximum velocity. It is clear that the maximum velocity U_M within the boundary layer gives a better boundary condition, at least for prediction of surface vorticity.

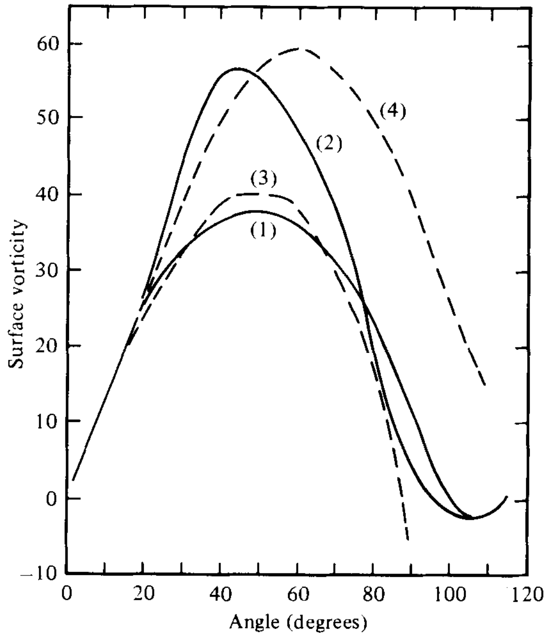

FIGURE 3. Surface vorticity. (1) This work. (2) Seeley *et al.* (3) Frossling, U_M. (4) Frossling, potential.

The value of U_M may be estimated from

$$U_M/U_0 = 1{\cdot}1959\theta - 0{\cdot}242906\theta^3 + 0{\cdot}0168130\theta^5$$

for $300 < Re < 3000$. The average deviation from experimental results is less than 10 %.

Another comparison with the results of Seeley *et al.* is presented in figure 3, which shows the measured surface vorticity made dimensionless with respect to the free-stream velocity $U_0 = 8{\cdot}87$ cm/s and the sphere radius $R = 3$ cm. The errors in the work of Seeley *et al.* induced higher values of the surface vorticity between 20° and 75° from the front stagnation point. Furthermore, they contributed to a displacement of the curve to the right. Subsequently, the curve did not extrapolate linearly to zero vorticity at $\theta = 0$. The surface vorticity of this work does extrapolate linearly to zero vorticity. Also, the separation angle θ_s measured in this work is higher than that reported by Seeley *et al.* For a Reynolds number $Re = 2940$, Seeley reported $\theta_s = 96°$, whereas the present results indicate that $\theta_s = 100°$. As has been shown, the present results, in general, are in good qualitative agreement with those of Seeley *et al.* However, an improved data-reduction technique has improved quantitative accuracy, and revealed a systematic phenomenon in the radial velocity profiles (curve 1, figure 2) which was not detected previously.

Lochiel & Calderbank (1964) developed a model for mass transfer around a sphere in which potential-flow theory was used to determine the outer boundary condition for the boundary layer. The tangential velocity at 90° from the front stagnation point calculated by the method of Lochiel & Calderbank is shown in figure 4 (curve 3). In the present work, numerical calculations suggested by Lochiel & Calderbank were carried out using the experimentally measured maximum tangential velocity U_M in the

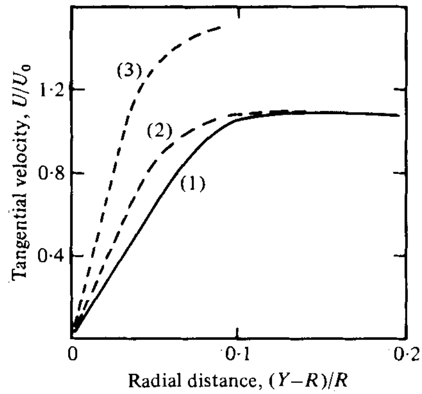

FIGURE 4. Tangential velocity at an angle of 90°. (1) This work. (2) Lochiel & Calderbank, U_M. (3) Lochiel & Calderbank, potential.

boundary layer in the boundary conditions. The result is shown in figure 4 (curve 2). These two sets of numerical solutions are compared with the result of the measured tangential velocity (curve 1). Once again, use of the maximum tangential velocity results in reasonable agreement with the measured velocity curve.

Conclusion

Photochromic-tracer films studying the dynamics of flow around spheres at intermediate Reynolds numbers have been analysed using a more reliable data-reduction technique and a simple data-processing method. The results of this analysis are, in general, in good qualitative agreement with the results obtained by Seeley *et al.* (1957). The present analysis leads to certain quantitative corrections to the data reported by these authors for intermediate Reynolds numbers. Comparison with some numerical solutions suggests that use of the maximum tangential velocity in the boundary layer as an outer boundary condition for the boundary-layer solution is a definite improvement over the use of the potential-flow solution

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